

Duration: two hours

5 points for each question.

Use of calculators and mobile phones is not allowed during the exam.

Evaluate the following integrals:

1. $\int \frac{\sqrt{x^2 - 9}}{x} dx$

2. $\int \frac{\cos x}{\sin x - 3 \csc x - 2} dx$

3. $\int x^3 \sec^2 x^2 dx$

Solve the following problems:

4. Let C be the curve with the parametrization: $x = 2t$ and $y = 2 \cosh t$, $t \in [0, \ln 2]$. Find the length of C .

5. Sketch the graphs of the polar equations $r_1 = 1 + \cos \vartheta$ and $r_2 = 1 - \cos \vartheta$. Find the area of the region that lies inside both graphs.

6. Show that the vectors $\mathbf{a} = \langle 2, 0, -1 \rangle$ and $\mathbf{b} = \langle 0, 1, 3 \rangle$ are not parallel to each other. Find a number t such that the vectors $\mathbf{u} = (1-t)\mathbf{a} + t\mathbf{b}$ and $\mathbf{v} = 3\mathbf{a} + 4\mathbf{b}$ are orthogonal.

7. Find the parametric equations of the line of intersection of the planes

$$(\pi_1): 2x - y + 2z + 5 = 0 \quad \text{and} \quad (\pi_2): x + 3y - 6z - 1 = 0$$

8. Let $f(x) = x^4 + x^3 + 1$, where $0 \leq x \leq 2$, and let $g(x) = f^{-1}(x)$. If $h(x) = f(2g(x))$, find $h'(3)$.